Reasoning About Programs

Mark Wheelhouse

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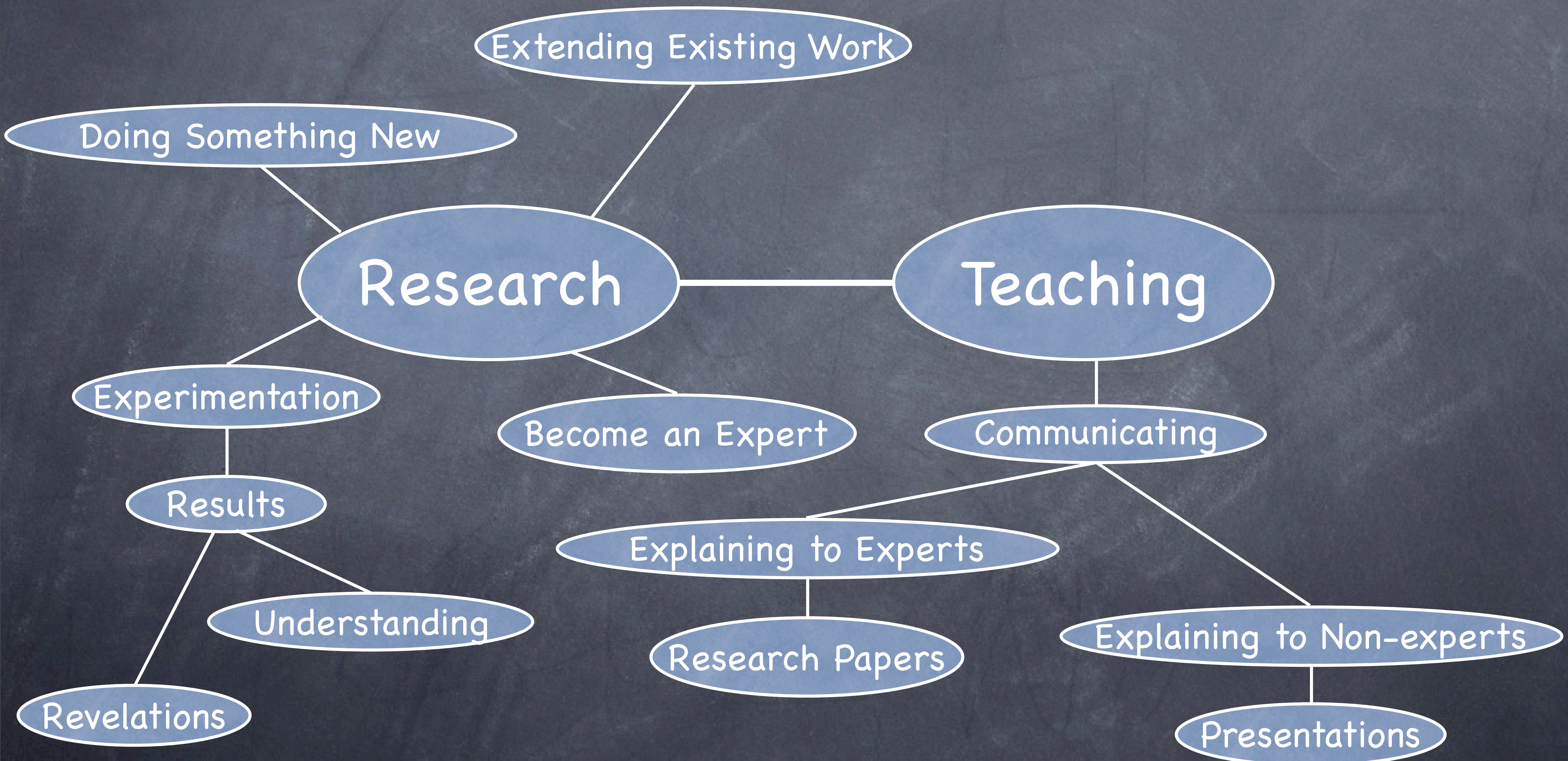
Department of Computing
Imperial College London

So What is a PhD Anyway?



What am I Doing?

"Context Logic, Tree Update and Concurrency"

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Huh?!

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What's He Talking About?!

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"Context Logic, Tree Update and Concurrency"

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What's He Talking About?!

Using Mathematics to prove properties about computer programs

Sometimes we want to be 100% sure the computer is doing what it should be

Why Prove Programs?
A History Lesson

Why Prove Programs? A History Lesson

1980's - Therac 25

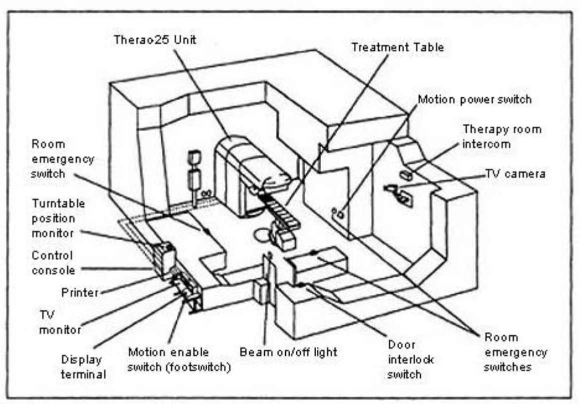

Figure 1. Typical Therac-25 facility

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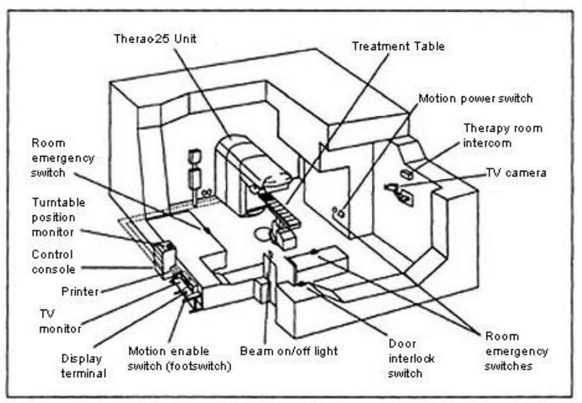

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race condition

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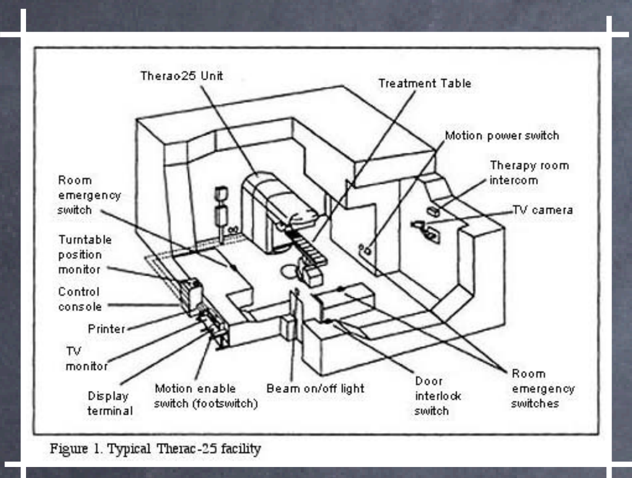

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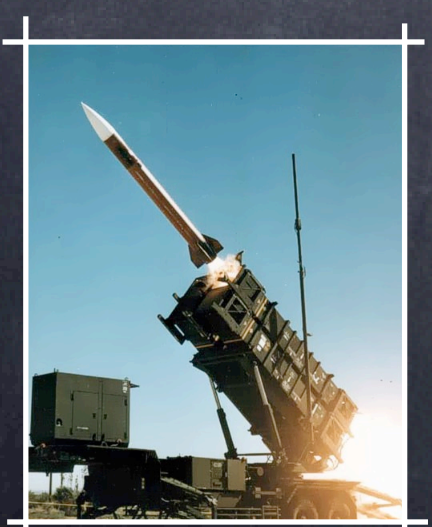
1991 - MIM-104 Patriot



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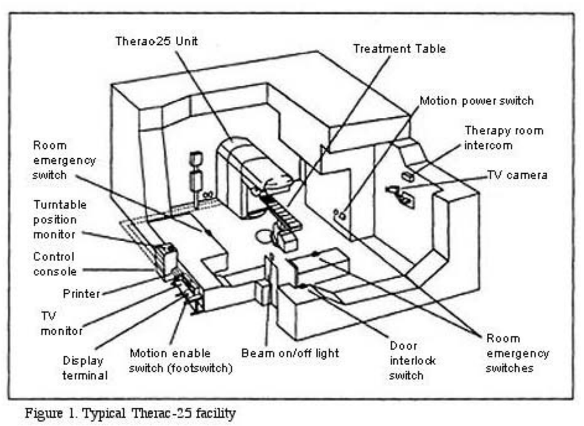

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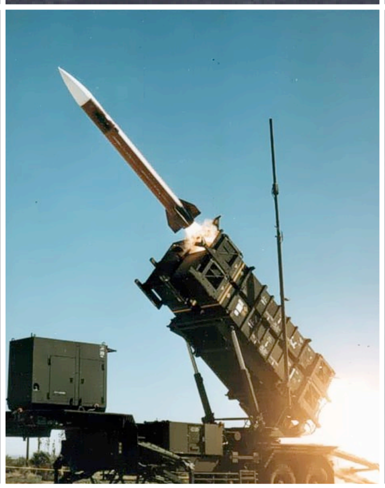

rounding error

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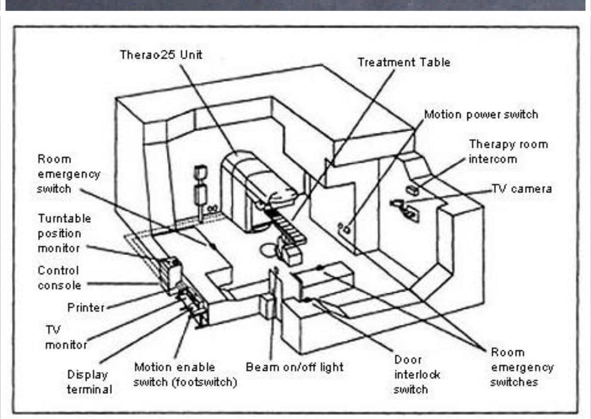

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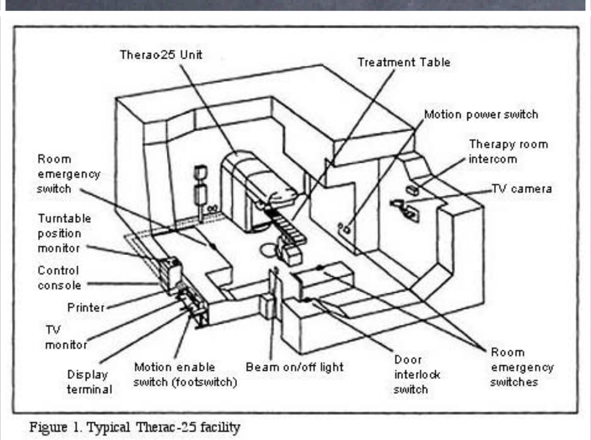

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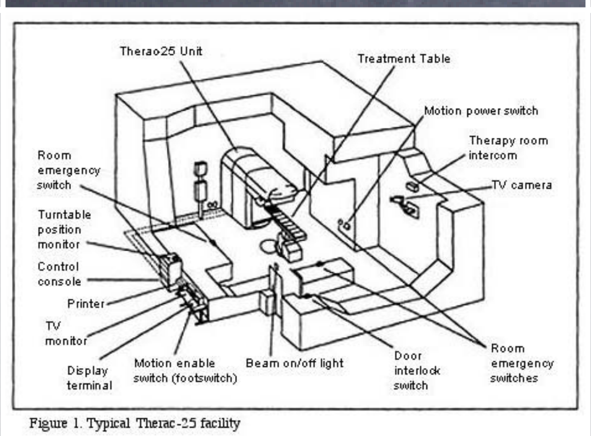

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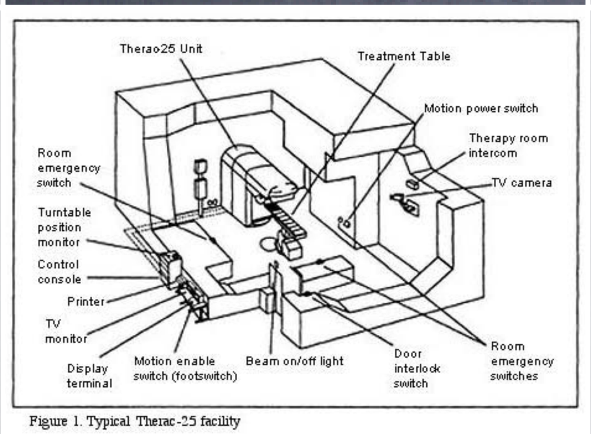

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divide by zero error!

What Kind of Systems do We Want to be Sure of?

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Internet Banking

Automatic Breaking

Nuclear Power Stations

Flight Control Systems

Building Design Software

IFF Targeting Systems

Heart Monitors

Spaceships

Pacemakers

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i.e. - Anything that is safety critical

- When the cost of system failure is huge

Spotting Failure

- We want to spot when a system can fail.
- We want to prove that a system will not fail.
- We can use mathematics (namely logic) to do this.

Logic - The Basics

boolean variables - “have one of two values”

P =	**true**	**false**
	1	0

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boolean operators - "ways of combining variables"

and: $\wedge$ or: $\vee$ implies: $\Rightarrow$

Logic - The Basics

boolean variables - "have one of two values"

	true	**false**
p =	1	0

boolean operators - "ways of combining variables"

and: $\wedge$

p	q	$p \wedge q$
0	0	0
0	1	0
1	0	0
1	1	1

or: $\vee$

p	q	$p \vee q$
0	0	0
0	1	1
1	0	1
1	1	1

implies: $\Rightarrow$

p	q	$p \Rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

Logic - Predicates

X,Y,Z are things (objects, people, concepts,...)

isRed(X) = true if X is red

isRound(X) = true if X is round

isRedBall(X) = true if X is a red ball

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isRedBall(X) = true if X is a red ball

$$isRedBall(X) \Rightarrow isRed(X) \wedge isRound(X)$$

Logic - Structures

another way of looking at this using a satisfaction relation,

$$\models$$

$X \models \text{isRed} \quad \Leftrightarrow \quad X \text{ is red}$

$X \models \text{isRound} \quad \Leftrightarrow \quad X \text{ is round}$

$X \models \text{isRedBall} \quad \Leftrightarrow \quad X \text{ is a red ball}$

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$$\text{isRedBall} \Rightarrow \text{isRed} \wedge \text{isRound}$$

Logic - Structures

data $\models$ Tree-Formula

tree T ::=

Logic - Structures

data $\models$ Tree-Formula

tree T ::= 0 empty tree

Logic - Structures

data $\models$ Tree-Formula

tree	T ::=	0	empty tree
		n[T]	tree node

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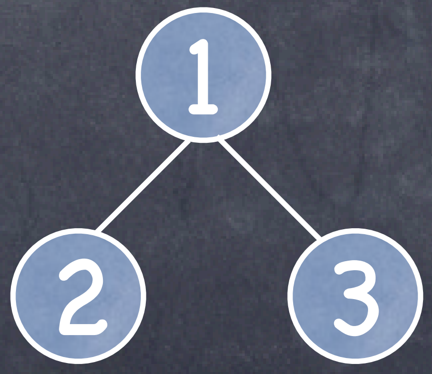
Eg: 1[2[0] | 3[0]]

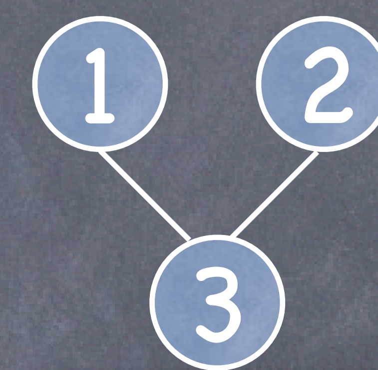


Tree Examples

a)

b)

c)



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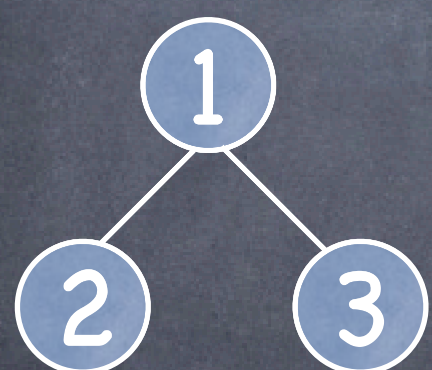

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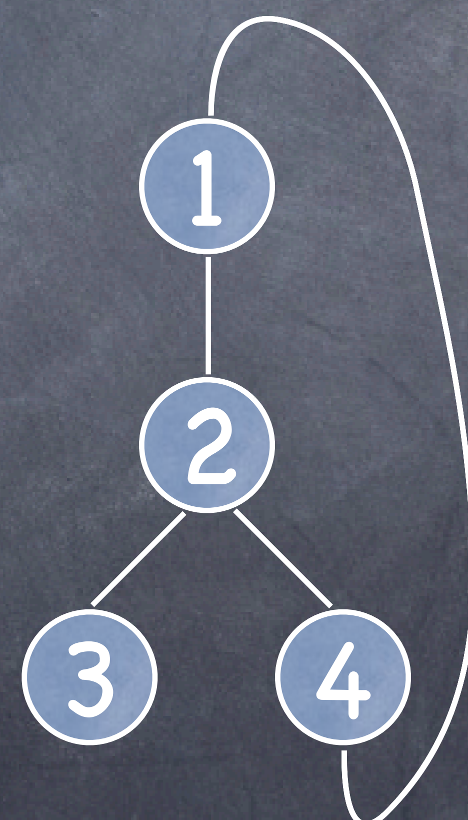

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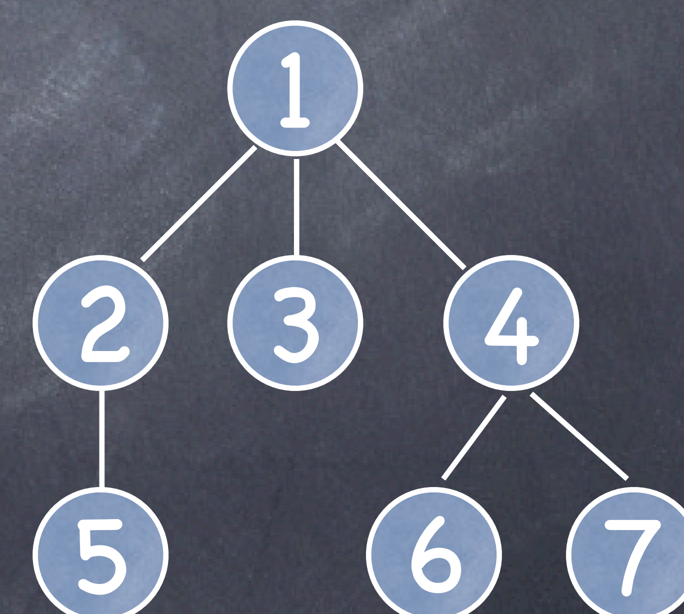

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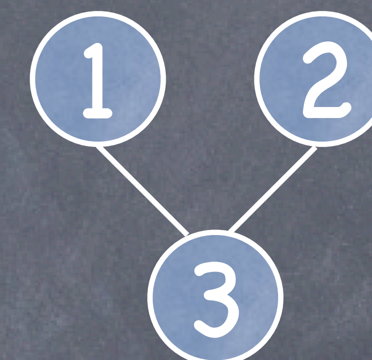

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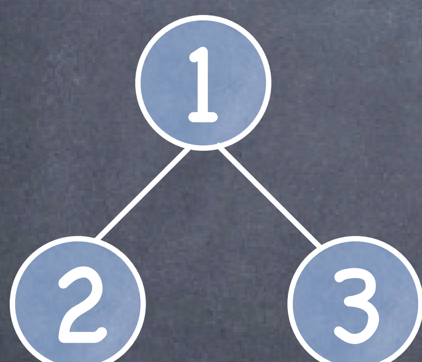

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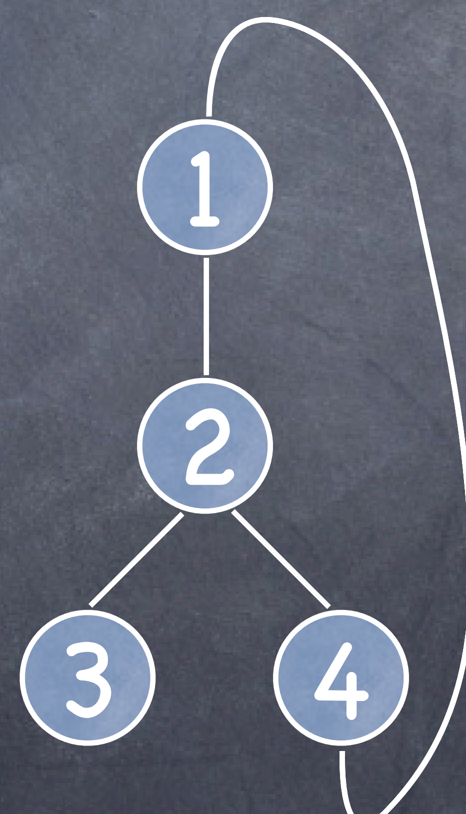

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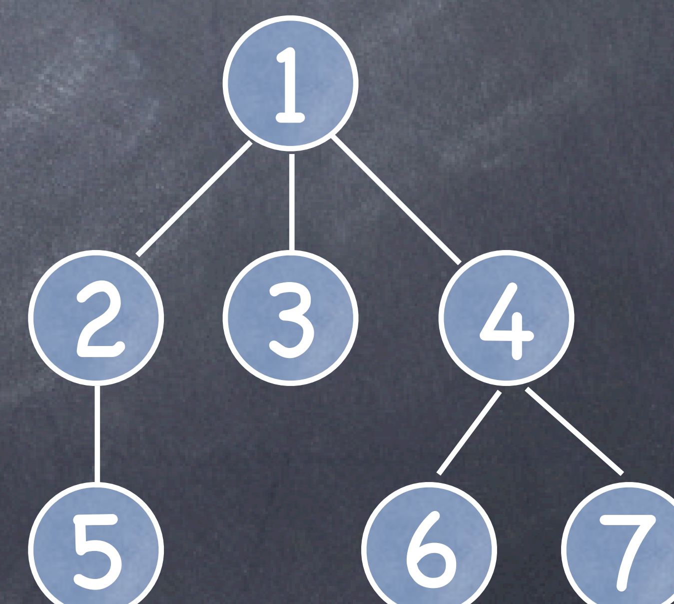

Tree Examples

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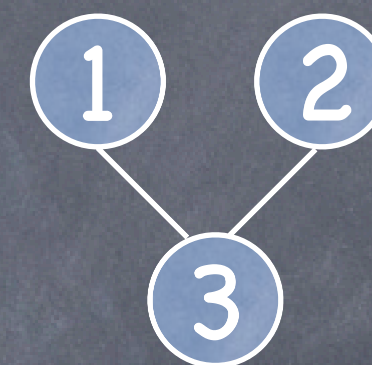
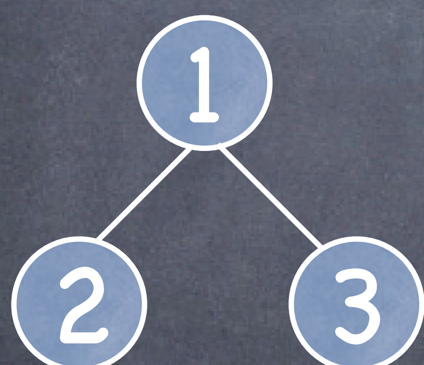
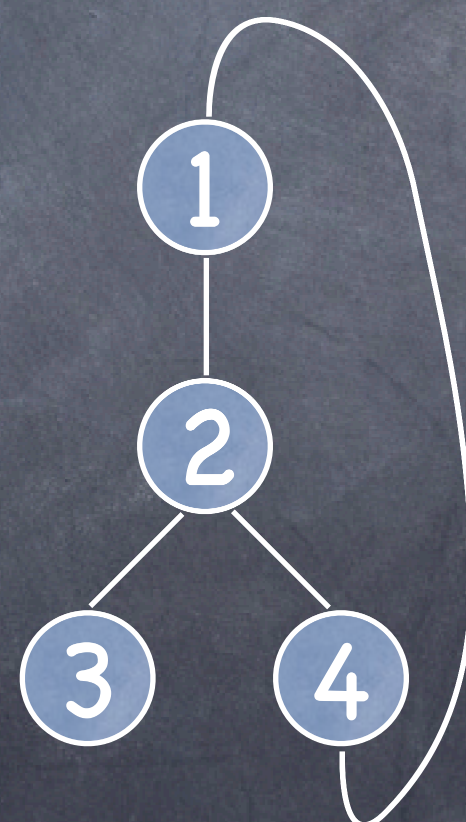
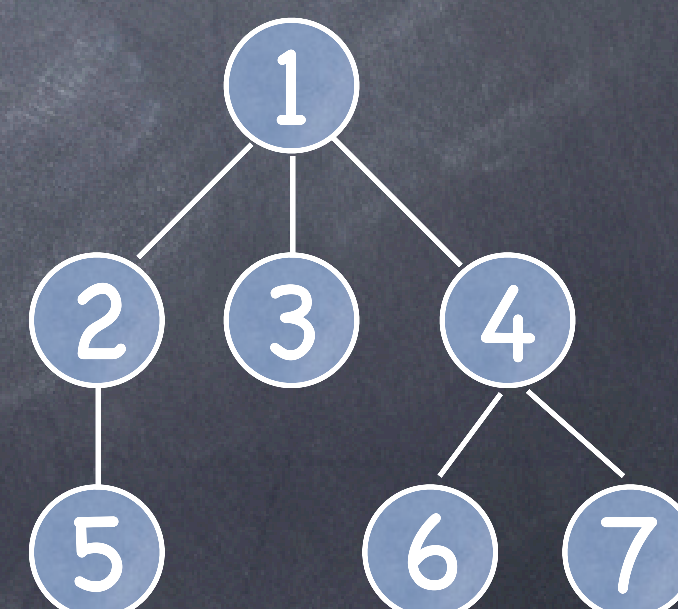
1[0] | 2[0]

c) 

d) 

e) 

f) 

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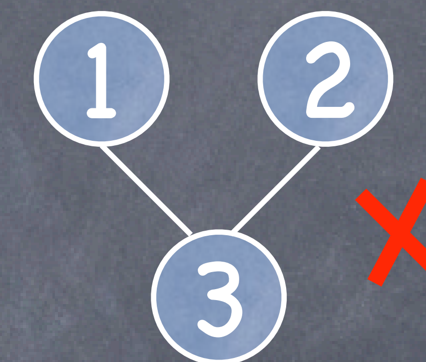

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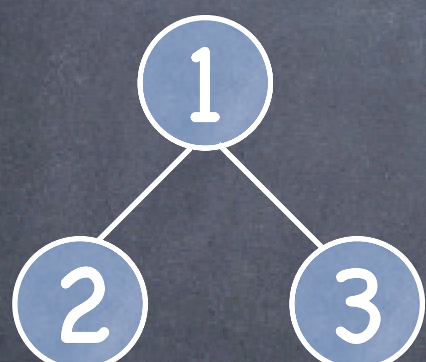

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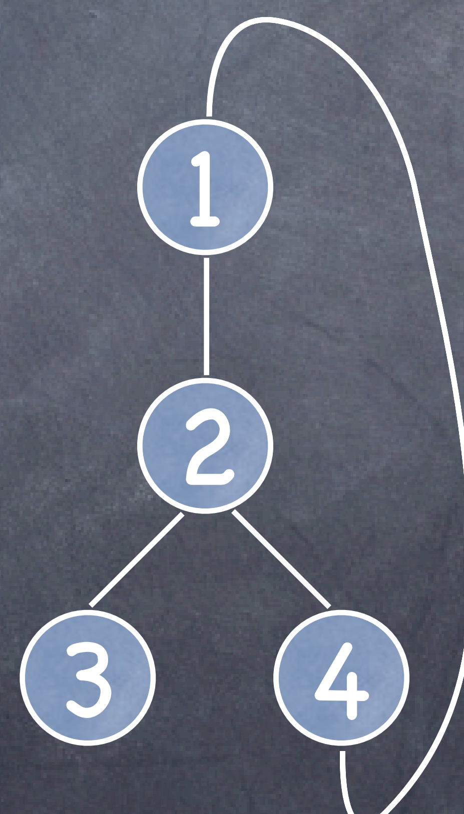

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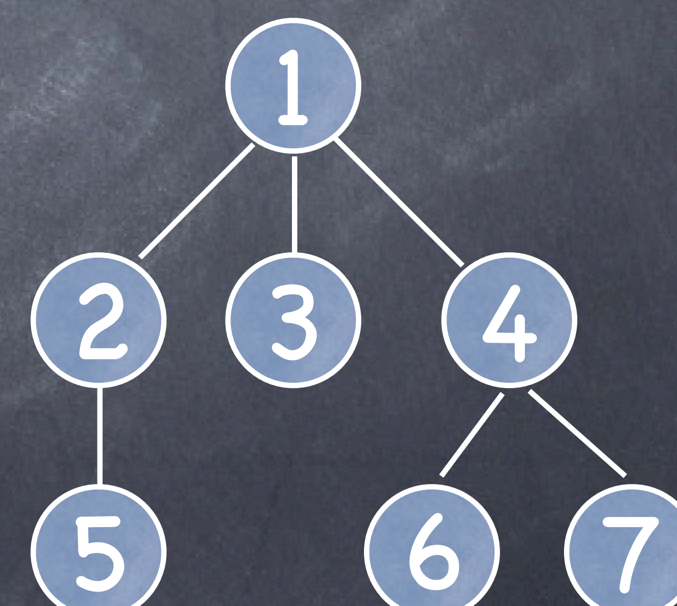

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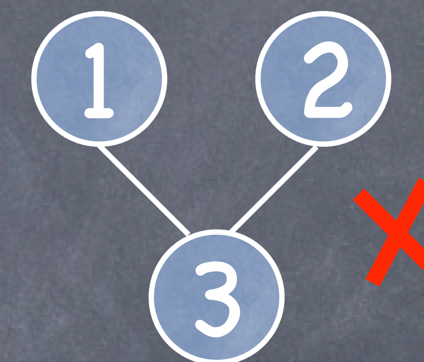
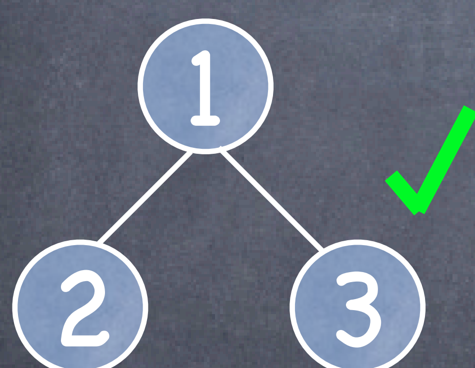
c) 

d) 

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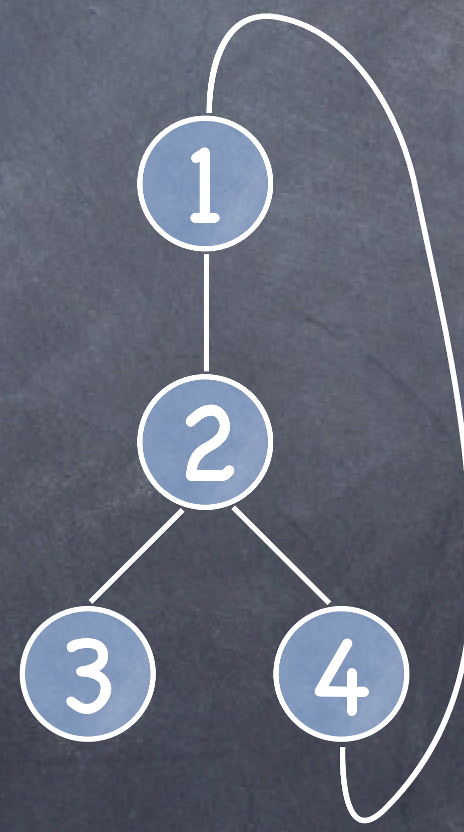
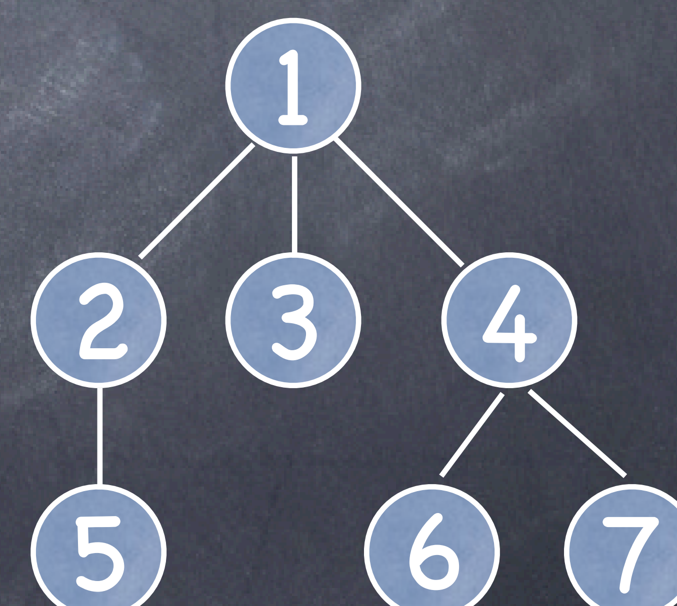
e) 

f) 

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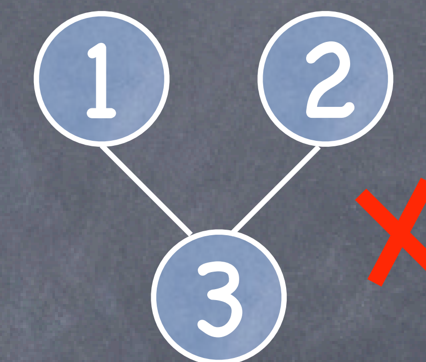

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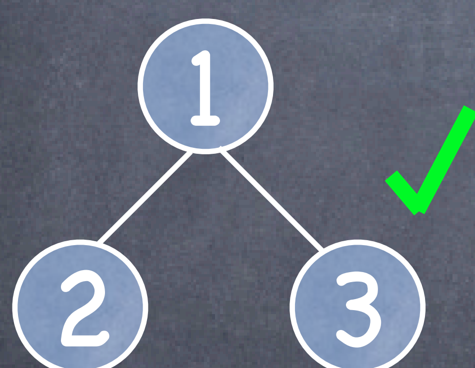

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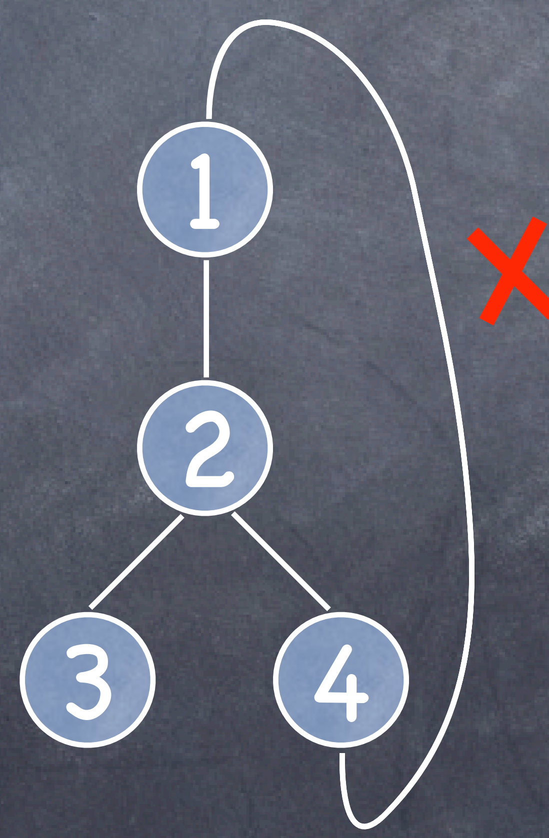

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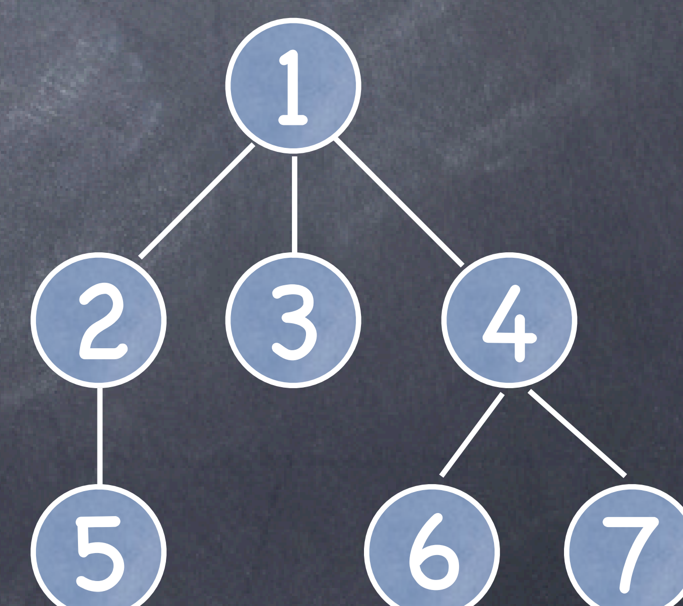

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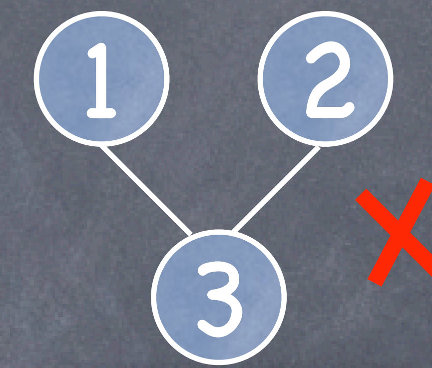

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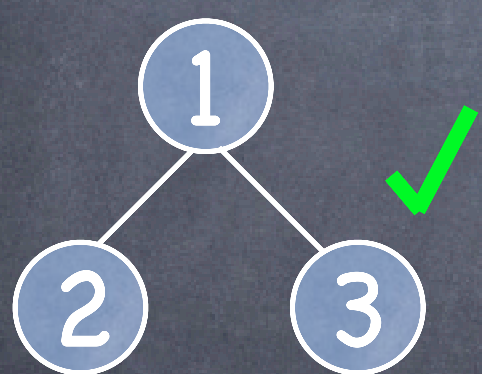

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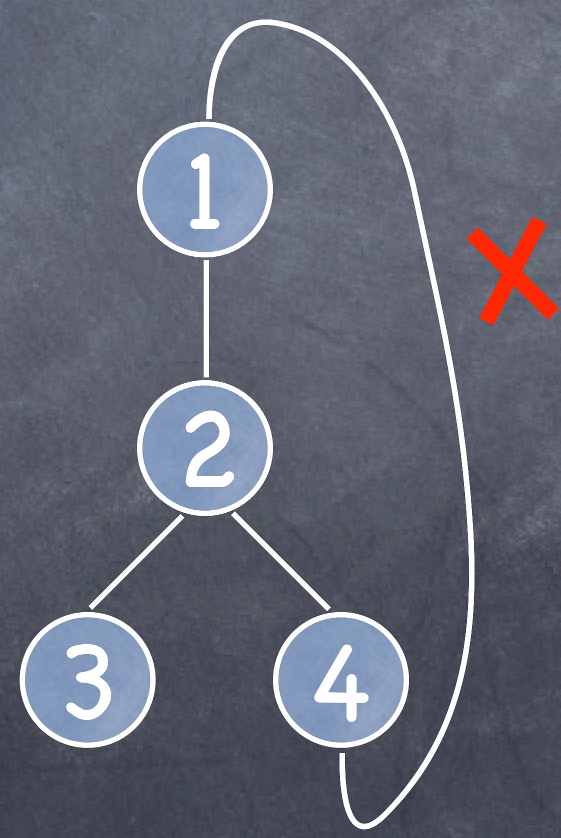

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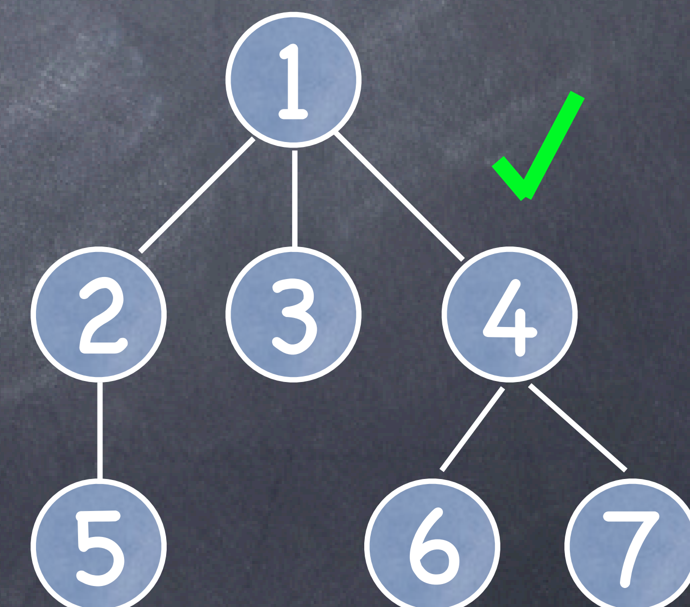

1[2[5[0]] | 3[0] | 4[6[0] | 7[0]]]

Data Update

We want to describe the behavior of a program,

Eg:

{ colour(X, ?) }

paint(X, Green)

{ colour(X, Green) }

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{ P } computer command { Q }

pre-condition post-condition

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Data Update

We want to describe the behavior of a program,

{ P } computer command { Q }

pre-condition post-condition

pre-condition must hold before and post-condition must hold after, or we get a fault!

Eg:

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paint(X, Green)

{ colour(X, Green) }

Local Data Update

$\{ y=3, x=n, z=2 \}$

addOne(x)

$\{ y=3, x=n+1, z=2 \}$

Local Data Update

{ y=3, x=n, z=2 }
addOne(x)
{ y=3, x=n+1, z=2 }

↓

{ x=n }
addOne(x)
{ x=n+1 }

Local Tree Update

$\{\ m[\ t_1 \mid n[t_2] \mid t_3\ ]\ \}$

p := goUpTree(n)

$\{\ m[\ t_1 \mid n[t_2] \mid t_3\ ] \wedge (p=m)\ \}$

$\{\ n[t]\ \}$

deleteTree(n)

$\{\ 0\ \}$

$\{\ n[t]\ \}$

addNodeAfter(n,x)

$\{\ n[t] \mid x[0]\ \}$

Example - Family Trees

A more complex data structure,

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A more complex data structure,

database D ::=

Example - Family Trees

A more complex data structure,

database D ::= 0_D empty database

Example - Family Trees

A more complex data structure,

database	D ::=	0_D	empty database
		familyName[F]	one family

Example - Family Trees

A more complex data structure,

database	$D ::=$	0_D	empty database
		familyName[F]	one family
		$D + D$	database join

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family tree $F ::=$

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database D ::=	0_D	empty database
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family tree F ::=	0_F	empty tree
	name:marTo Gen:Age [F]	tree node

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A more complex data structure,

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	F | F	ordered trees

Example - Family Tree Nodes

name:marTo
Gen:Age [F] tree node

Example - Family Tree Nodes

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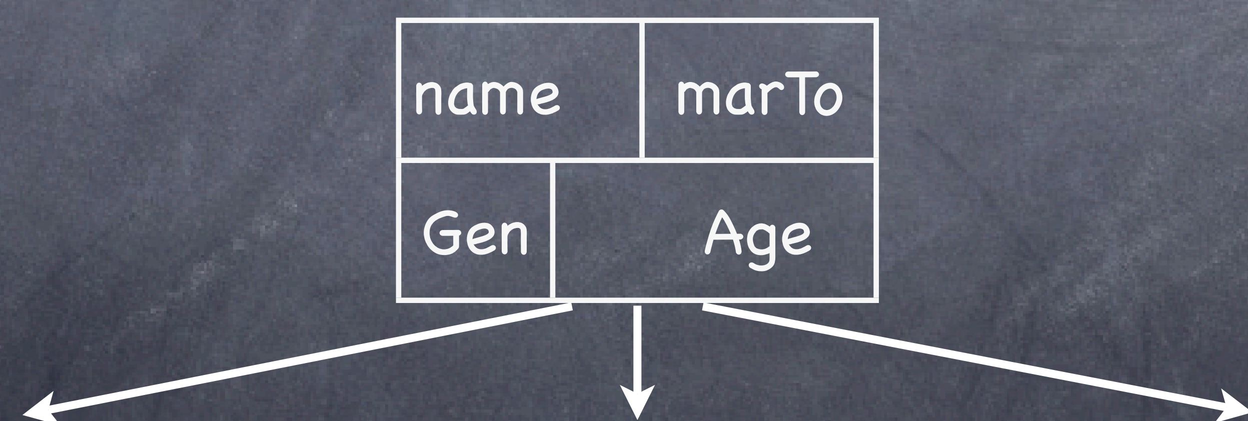
II

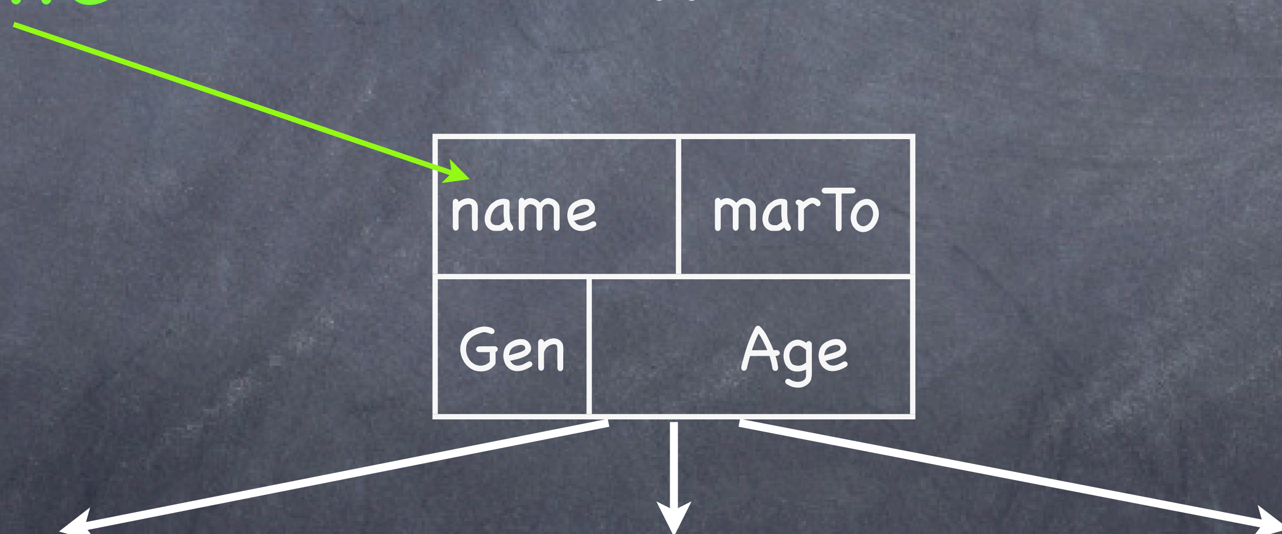


Example - Family Tree Nodes

name:marTo
Gen:Age [F] tree node

||

person's name



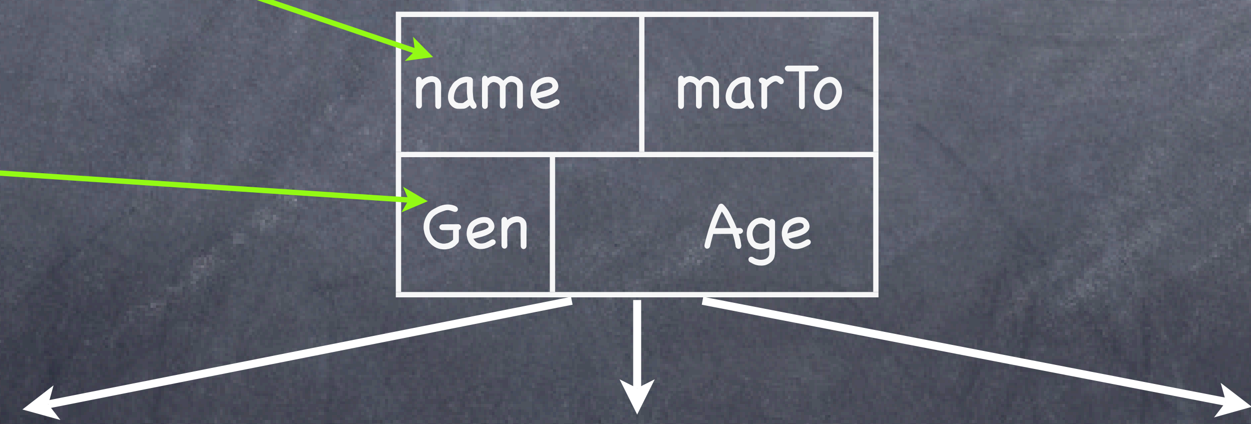
Example - Family Tree Nodes

name:marTo
Gen:Age [F] tree node

=

person's name

person's gender (M or F)



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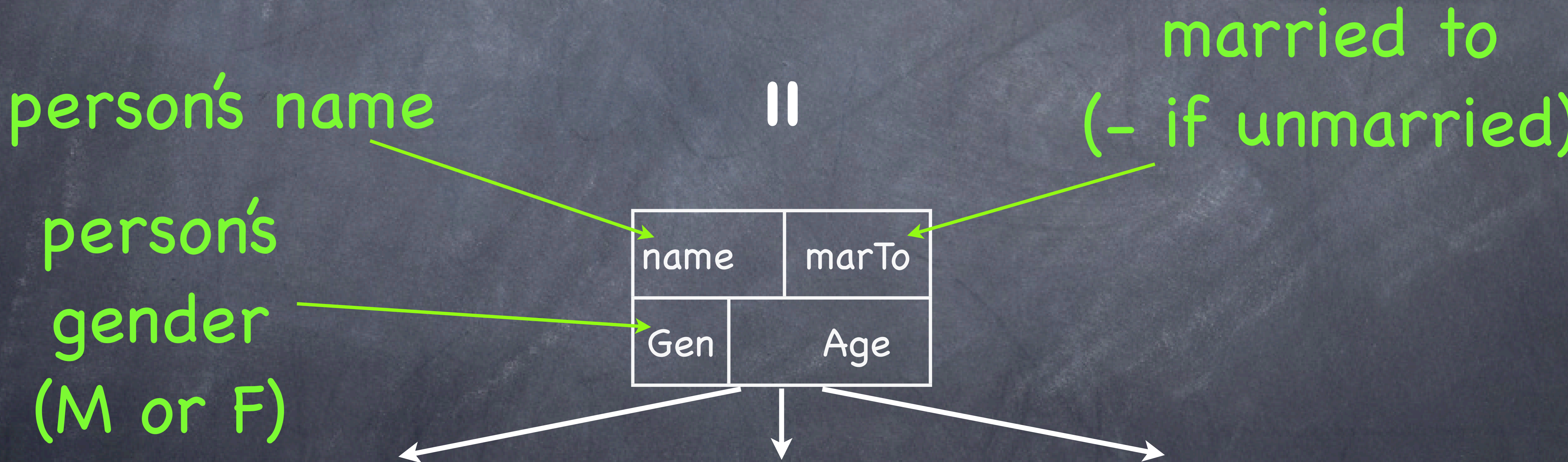

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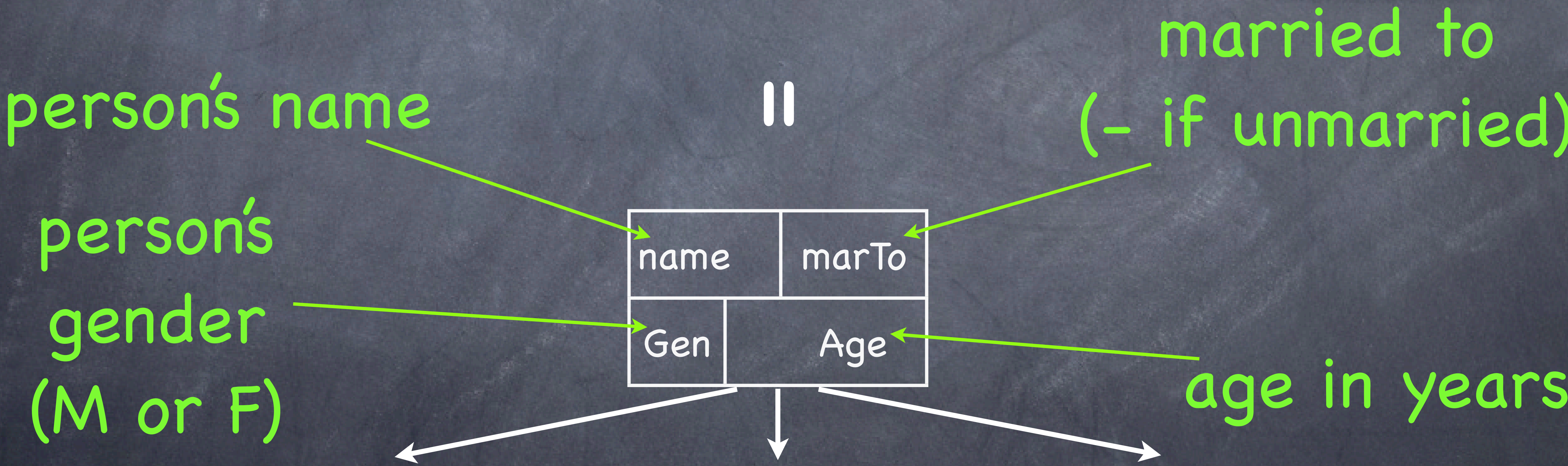

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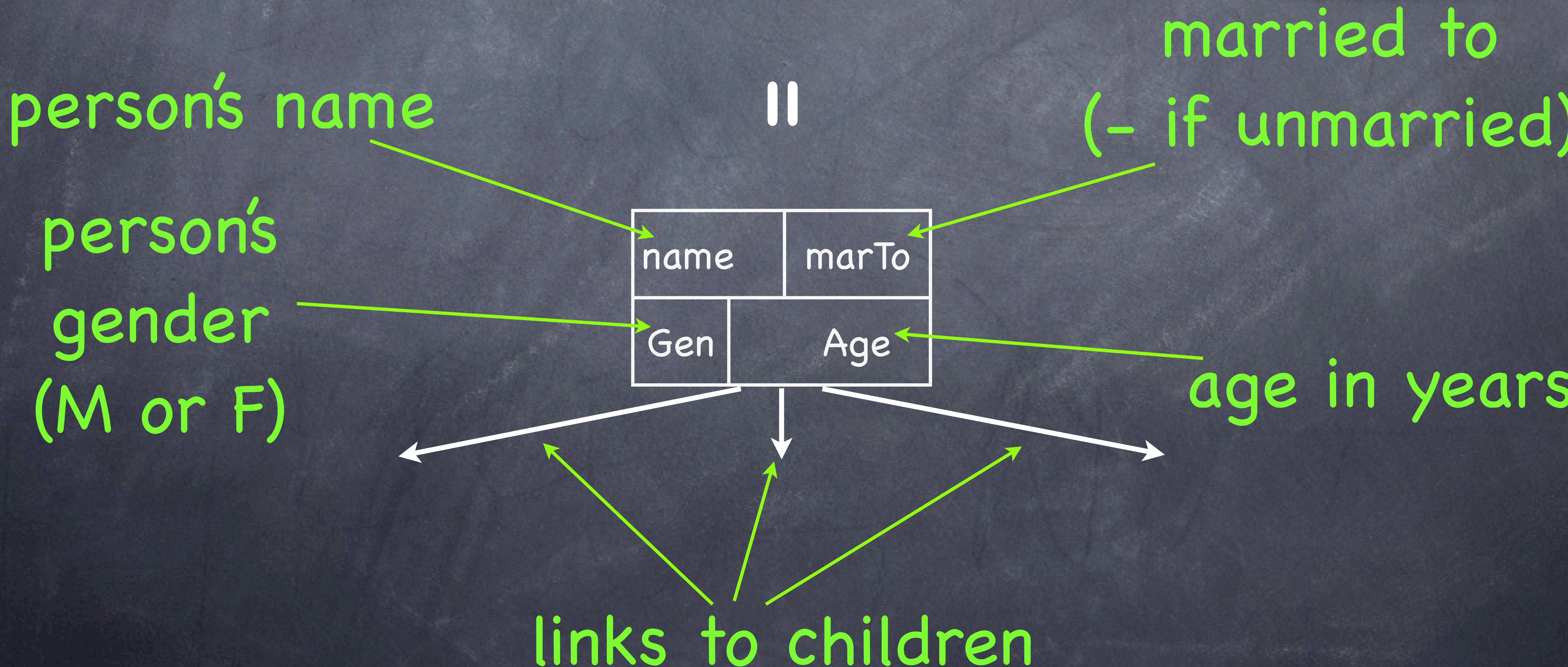
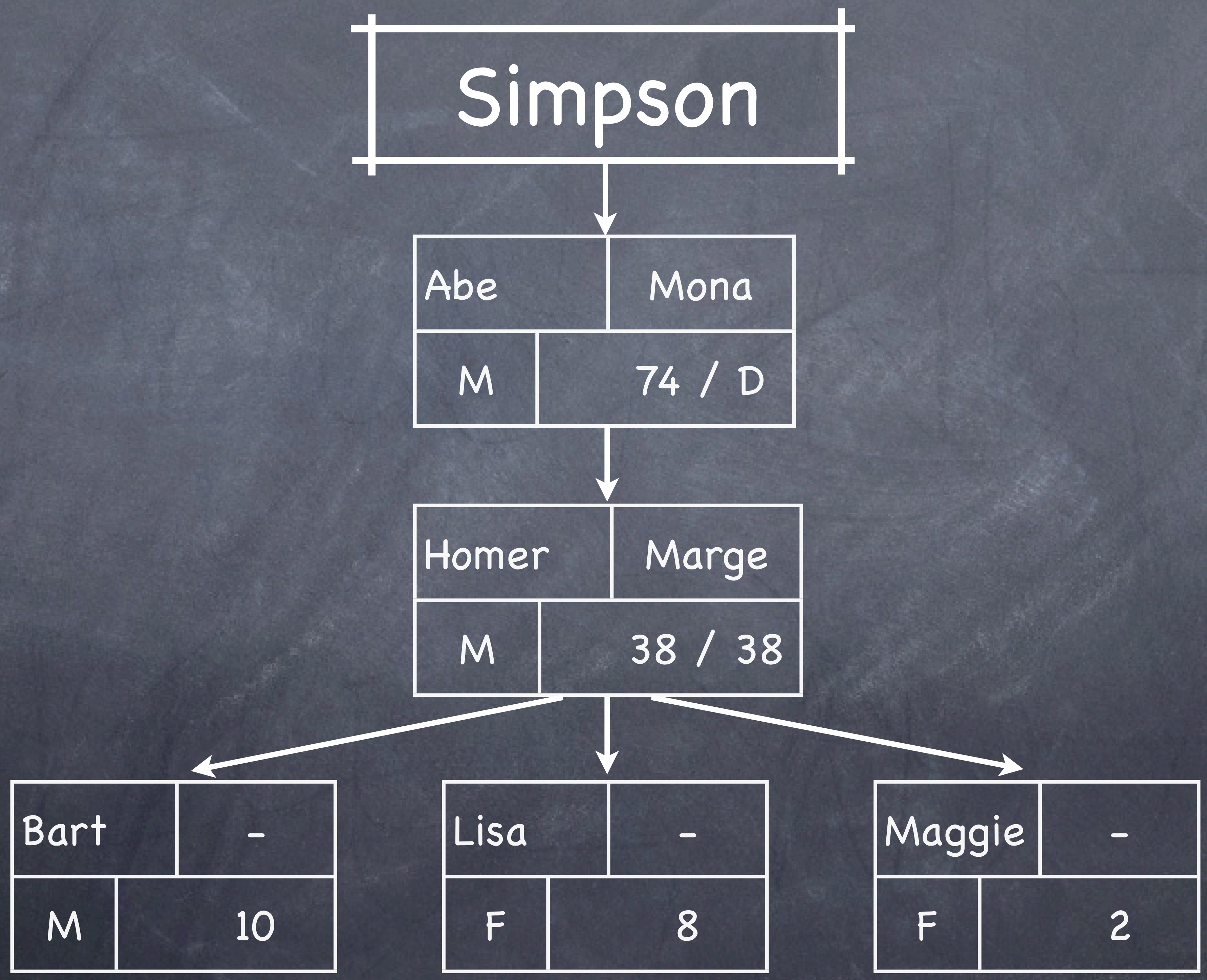
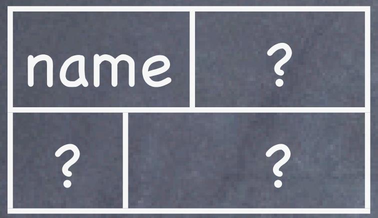


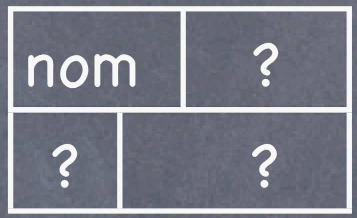
Example - The Simpsons



Example - Commands

Example - Commands

{ 

} reName(name, nom) { 

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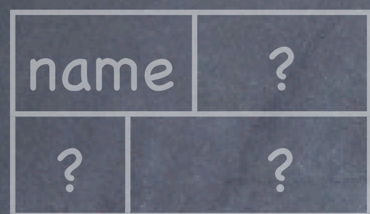
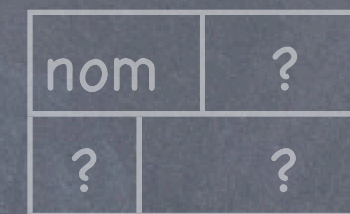
Example - Commands

{ 

} reName(name, nom) { 

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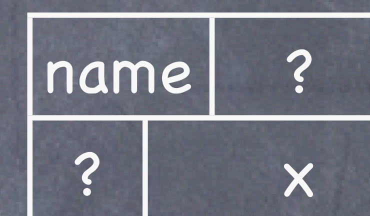
} reage(name, x) { 

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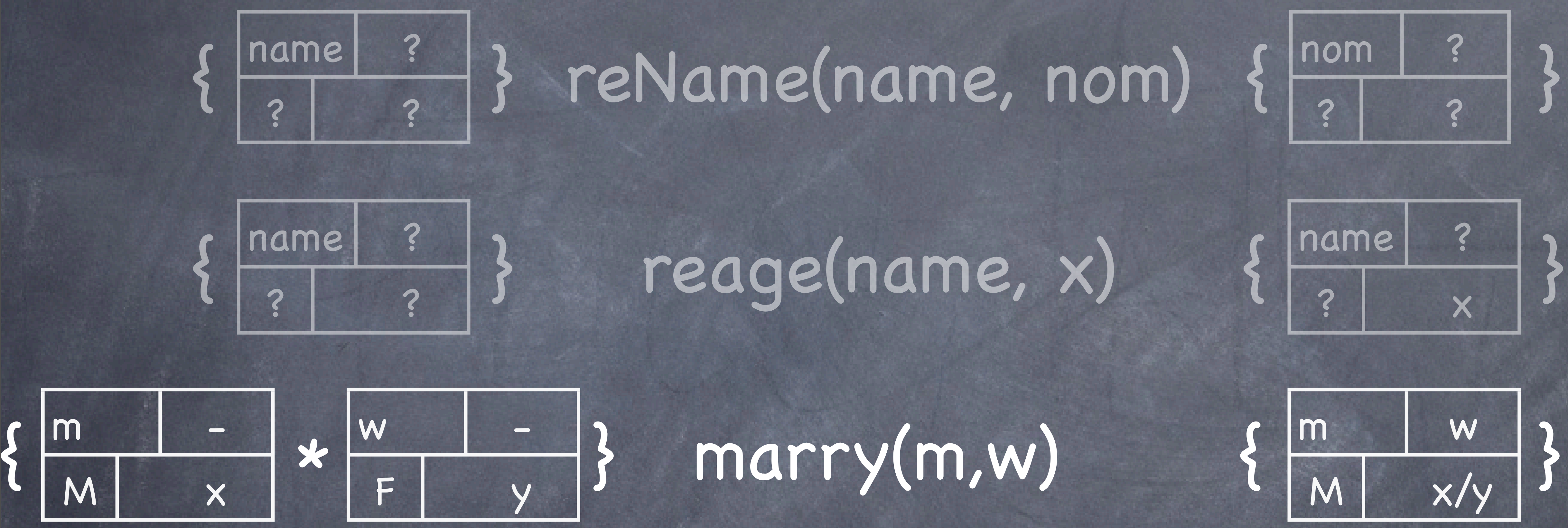
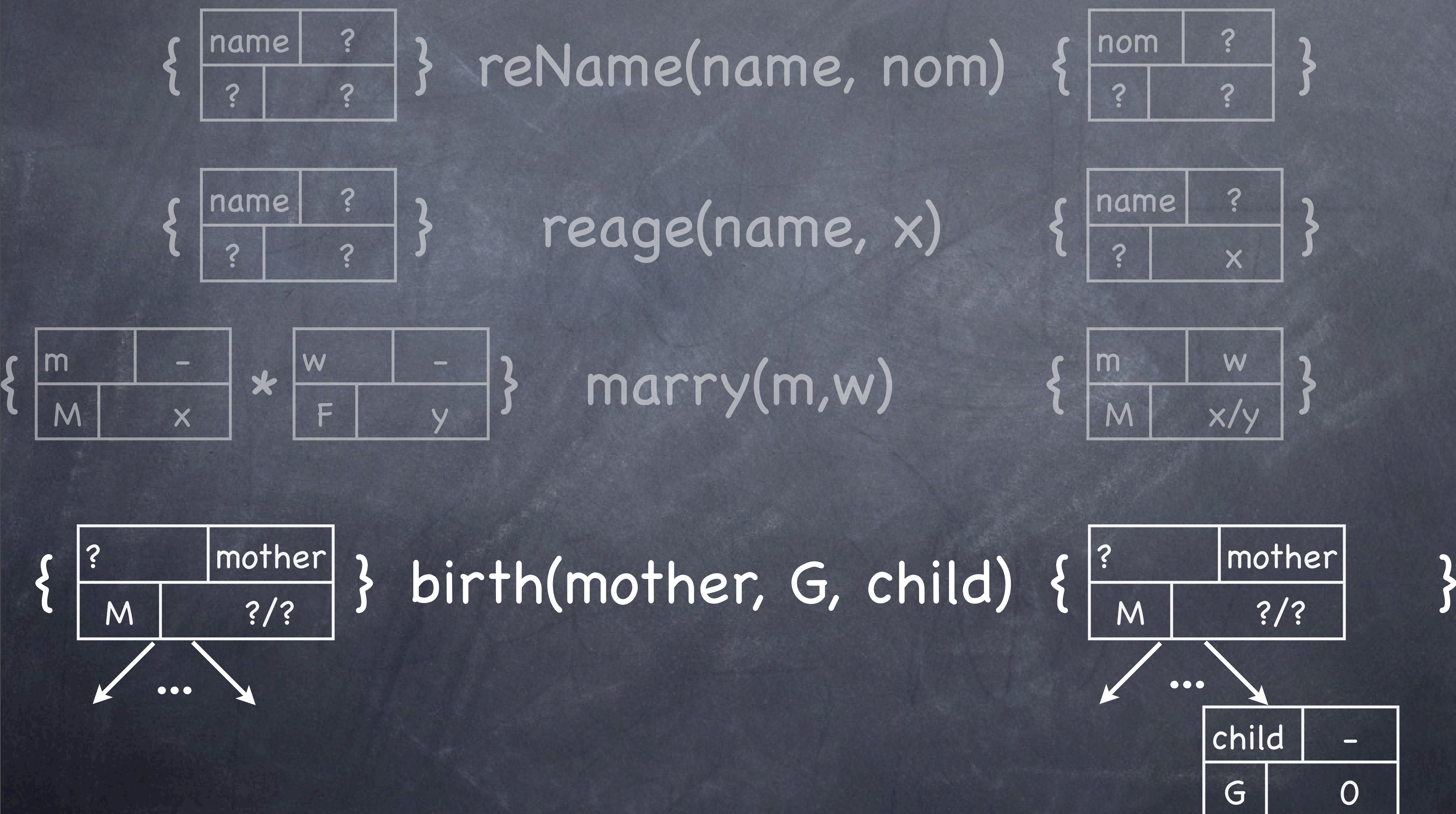
Example - Commands



Example - Commands



Now you are going to perform concurrent local reasoning!

Example - Commands